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2008 J. Phys.: Condens. Matter 20 474214

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J. Phys.: Condens. Matter **20** (2008) 474214 (5pp) doi:10.1088/0953-8984/20/47/474214

Spin separation driven by quantum interference in ballistic rings

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Received 23 April 2008, in final form 15 May 2008
Published 6 November 2008
Online at stacks.iop.org/JPhysCM/20/474214

Abstract
We propose an all-electrical nanoscopic structure where a *pure spin current* is induced in the transverse probes attached to a *quantum-coherent* ballistic quasi-one-dimensional ring when conventional unpolarized charge current is injected through its longitudinal leads. The study is essentially based on the spin–orbit coupling (SOC) arising from the laterally confining electric field (β-SOC). This sets the basic difference with other works employing mesoscopic rings with the conventional Rashba SO term (α-SOC). The β-SOC ring generates oscillations of the predicted spin Hall current due to *spin-sensitive quantum-interference effects* caused by the difference in phase acquired by opposite spins states traveling clockwise and counterclockwise. We focus on single-channel transport and solve analytically the spin polarization of the current. We relate the presence of a polarized spin current with the peaks in the longitudinal conductance.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Conventional electronic devices rely on the transport of electrical charge carriers—electrons—in a semiconductor. Now physicists are trying to exploit the 'spin' of the electron rather than its charge, in order to create a remarkable new generation of 'spintronic' devices [1, 2]. Current efforts in spintronics are directed towards gaining control of the electron spin in submicrometric devices. Thus, some fundamental quantum phenomena that involve electron spin have arisen, in order to generate and measure *pure spin currents*. These currents emerge when an equal number of spin-$\uparrow$ and spin-$\downarrow$ electrons move in opposite directions, so that the net charge current is zero [3].

The so called *spin Hall effect* (SHE) has been extensively investigated in recent years, in order to obtain a pure spin current. The classic Hall effect occurs when an electric current flows through a conductor subjected to a perpendicular magnetic field. In this case the Lorentz force deflects the electrons and charge builds up on one side of the conductor, resulting in an observable Hall voltage [4]. In the absence of an external magnetic field, some esoteric Hall-type effects involving electron spin become possible in systems with spin–orbit (SO) interactions, such as the SHE [5, 6]. In these systems, as an extension of Ehrenfest's theorem in quantum mechanics, the SO coupling may generate a spin-dependent transverse force on moving electrons [7]. This force tends to separate different spins in the transverse direction as a response to the longitudinal charge current, giving a qualitative explanation for the SHE.

Thus, the SO coupling plays a central role in the SHE phenomenology and its properties were largely investigated in two-dimensional electron systems (2DESs). In the case of quantum heterostructures of narrow gap semiconductors, a major contribution to the SO coupling may originate intrinsically from the asymmetry in the quantum well potential that confines the electron gas in a 2D plane (Rashba or α-SO coupling) [9]. However, in quasi-one-dimensional (Q1D) devices patterned in a 2DES, such as quantum wires, a confining SO coupling (β-coupling) arises from the in-plane electric potential that is applied to squeeze the 2DES into a Q1D channel [10, 11].

In some recent papers [12–14] the SHE like phenomenology due to the β coupling was discussed, and it was shown that the SOC could give stronger effects than the α one. In some devices, such as Q1D wires [12], the effect of the β-SO term is analogous to the one of a uniform effective magnetic field, $B_{\rm eff}$, orthogonal to the 2DES (x–y plane) directed upwards or

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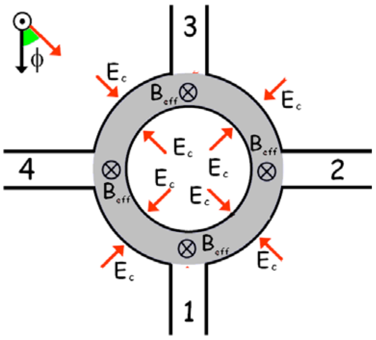

Figure 1. The mesoscopic circuit serving as a generator of the pure ($I_2 = I_2^\uparrow + I_2^\downarrow = 0$) spin Hall current $I_2^s = \frac{\hbar}{2e}(I_2^\uparrow - I_2^\downarrow) = -I_4^s$ in the transverse voltage probes ($V_2 = V_4 \neq 0$, $I_2 = I_4 = 0$) attached to a ring realized using a 2DEG in a semiconductor heterostructure [8]. The injected unpolarized ($I_1^s = 0$) current through (single-channel) longitudinal leads is subjected to the β SO interaction (nonvanishing in the shaded ring region), which acts as a field $B_{\rm eff}$ arising due to the electric field $\mathbf{E}_c$ confining the electrons in a quasi-one-dimensional (Q1D) ring. The angle φ, the coordinate in the ring's regions, vanishes at the intersection with lead 1, whereas it takes the value $\varphi = \pi/2$ at the intersection with lead 2, and so on.

downwards according to the spin polarization along the z direction. Thus, the SOC exerts a spin-dependent transverse force on moving electrons, while conserving their spins.

In this paper we focus on the aspects of spin interference in ballistic Q1D ring geometries with four leads subject to β-SO coupling (see figure 1). In fact, ring conductors smaller than the dephasing length $L_\phi \lesssim 1\ \mu$m (at low temperature $T \ll 1$ K) have played an essential role in observing how *coherent superpositions* of quantum states (i.e., quantum-interference effects) on a mesoscopic scale leave an imprint on measurable transport properties. These ballistic rings represent a solid-state realization of a two-slit experiment—an electron entering the ring can propagate in two possible directions (clockwise and counterclockwise) where superpositions of corresponding quantum states are sensitive to the acquired topological phases [15] in magnetic (Aharonov–Bohm (AB) effect) or electric (Aharonov–Casher (AC) effect for particles with spin) external field whose variation generates an oscillatory pattern of the ring conductance [8]. Many recent papers proposed the mesoscopic quantum ring, as a device able to select a spin polarized current, focusing on the role of the α coupling [16–21].

2. The SO coupling

The SO interaction is generally described by the Hamiltonian [22]

$$\hat{H}_{\rm SO} = -\frac{\lambda_0^2}{\hbar}\, e\mathbf{E}(\mathbf{r})\left[\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}\right]. \qquad (1)$$

Here $\mathbf{E}(\mathbf{r})$ is the electric field, m_0 the electron mass in vacuum, $\hat{\sigma}$ are the Pauli matrices, $\hat{\mathbf{p}}$ is the canonical momentum operator, $\mathbf{r}$ is a three-dimensional position vector and λ_0^2 is the SO coupling parameter having a dimension of length squared. In materials m_0 and λ_0 are replaced by their effective values m^* and λ. Next, we neglect the α term of SO coupling and adapt the general form to the strictly 2D case, where the degree of freedom for the motion in the z direction is frozen out (i.e. with a mean value $\langle p_z\rangle = 0$ in the ground state, for the potential well in the z direction), and the potential energy depends only on x and y coordinates. Then $\hat{H}_{\rm SO}$ takes the following form [23]:

$$\hat{H}^\beta_{\rm SO} = \frac{\lambda^2}{\hbar}\hat{\sigma}_z\left[\nabla V_{\rm c}(\mathbf{r}) \times \hat{\mathbf{p}}\right]_z, \qquad (2)$$

where $V_{\rm c}(r)$ is the 2D confining potential.

3. SO interaction in quasi-one-dimensional systems

The basic building block of our device is the ballistic one-dimensional wire i.e. a nanometric solid-state device in which the transverse motion (along x) is quantized into discrete modes, and the longitudinal motion (y direction) is free. In this case electrons are envisioned as propagating freely down a clean narrow pipe and electronic transport with no scattering can occur.

In line with [24], the lateral confining potential of a QW, $V_{\rm c}(x)$, is approximated by a parabola $V_{\rm c}(\mathbf{r}) = \frac{m^*}{2}\omega^2 x^2$. The quantity ω controls the strength (curvature) of the confining potential while the in-plane electric field $e\mathbf{E}_c(\mathbf{r}) = -\nabla V_c(\mathbf{r})$ is directed along the transverse direction. For the parabolic confining potential $\hat{H}^\beta_{\rm SO}$ is

$$\hat{H}^\beta_{\rm SO} = \frac{\beta}{\hbar}\frac{x}{l_\omega}\left(\hat{\sigma} \times \hat{\mathbf{p}}\right)_x = {\rm i}\beta\frac{x}{l_\omega}\sigma_z\frac{\partial}{\partial y}. \qquad (3)$$

Here $l_\omega = (\hbar/m^*\omega)^{1/2}$ is the typical spatial scale associated with the potential V_c and $\beta \equiv \lambda^2 m^*\omega^2 l_\omega$. It follows that the effect of the β-SOC is analogous to the one of a uniform effective field,

$$B_{\rm eff} = \frac{\lambda^2}{\hbar}\frac{m^{*2}\omega^2 c}{e} \equiv \frac{\beta}{\hbar l_\omega}\frac{m^* c}{e}, \qquad (4)$$

orthogonal to the 2DEG directed upwards or downwards, according to the spin polarization along the z direction.

Next, we introduce the effective cyclotron frequency $\omega_{\rm c} = \frac{\beta}{\hbar l_\omega}$ ($\omega_{\rm c}/\omega = \lambda^2/l_\omega$), the related frequency $\omega_0^2 = \omega^2 - \omega_{\rm c}^2$ and the total frequency $\omega_{\rm T} = \sqrt{\omega^2 + \omega_{\rm c}^2}$, thus

$$\hat{H}_0 + \hat{H}^\beta_{\rm SO} = \frac{\omega_0^2}{\omega_{\rm T}^2}\frac{p_y^2}{2m^*} + \frac{p_x^2}{2m^*} + \frac{m^*\omega_{\rm T}^2}{2}(x - x_0)^2, \qquad (5)$$

where $x_0 = s\frac{\omega_{\rm c} p_y}{\omega_{\rm T}^2 m^*}$, $s = \pm 1$, corresponds to the spin polarization along the z direction. Hence we can conclude that 4-split channels are present for a fixed Fermi energy, $\varepsilon_{\rm F}$, corresponding to $\pm p_y$ and $s_z = \pm 1$.

4. Q1D ring

Here we outline briefly the derivation of the Hamiltonian describing the motion of an electron in a realistic Q1D ring [25]. We consider the 2DEG in the xy plane; then we introduce a radial potential $V_{\rm c}(r)$, so that the electrons are confined to move in a ring. To be specific, we use a parabolic radial confining potential $V_c(r) = \frac{1}{2}m^*\omega_d^2(r-R_0)^2$, for which the radial width of the wavefunction is given by l_ω. Due to the circular symmetry of the problem, it is natural to rewrite the full single-electron Hamiltonian in polar coordinates [25]

$$H = -\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\left(\mathrm{i}\frac{\partial}{\partial\varphi}\right)^2\right]+V_c(r) + \frac{\lambda^2}{\hbar}\mathrm{e}\frac{E_r(r)}{r}\left(-\mathrm{i}\hbar\frac{\partial}{\partial\varphi}\right)\sigma_z, \qquad (6)$$

because the electric field has just the radial component. It follows that $L_z = -\mathrm{i}\hbar\frac{\partial}{\partial\varphi}$ and σ_z commute with the Hamiltonian $\hat{H}$ and the corresponding eigenvalues are $\pm\hbar\mu$ for L_z and ±1 for σ_z.

In the case of a thin ring, i.e., when the radius R_0 of the ring is much larger than the radial width of the wavefunction, it is convenient to project the Hamiltonian on the eigenstates of

$$H_0 = -\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right]+V_c(r).$$

Next, we assume $l_\omega/R_0 \ll 1$ and neglect contributions of order l_ω/R_0 to H_0 and to the centrifugal term,

$$H_c \simeq -\frac{\hbar^2}{2m^*R_0^2}\frac{\partial^2}{\partial\varphi^2} = \hbar\omega_R\frac{\partial^2}{\partial\varphi^2}.$$

After some tedious calculations [26] we are able to obtain the energy spectrum as

$$\varepsilon_{n,\mu,s} \sim \hbar\sqrt{\omega^2+2\omega_c\omega_R\mu s}(n+\tfrac{1}{2})+\hbar\omega_R\mu^2. \qquad (7)$$

The corresponding bandstructure is shown in figure 2. It follows that for fixed values of the Fermi energy, ε_F, and of the band n there are 4 different eigenstates $\Psi^s_{n,\mu}$ i.e. particles with fixed Fermi energy ε_F can go through the ring with four different wavenumbers $\pm\mu_{\pm,s}$, depending on spin (s) and direction of motion ($\pm$). Moreover the presence of a nonvanishing β term implies an edge localization of the currents depending on the electron spins, also giving the presence of two localized spin currents with opposite chiralities [12].

The presence of a spin splitting is the basis of the interference phenomena in the transport through the ring. By solving $\varepsilon_{\mu,n} = \varepsilon_F$ we can obtain the values of $\mu_{\pm,s}$ that are not required to be integer, whereas $\mu_{+,\uparrow} = \mu_{-,\downarrow}$ and $\mu_{-,\uparrow} = \mu_{+,\downarrow}$, because of the symmetry of the system. Thus, a momentum difference arises

$$\Delta\mu = \mu_{+,\uparrow}-\mu_{-,\uparrow} = \mu_{-,\downarrow}-\mu_{+,\downarrow} \sim 2\frac{\omega_c}{\omega}.$$

5. Quantum transport of spin currents in four-terminal rings

The charge currents in mesoscopic structures attached to many leads are described by the multiprobe Landauer–Büttiker formulas [27]

$$I_p = \sum_{q\neq p} G_{pq}(V_p - V_q), \qquad (8)$$

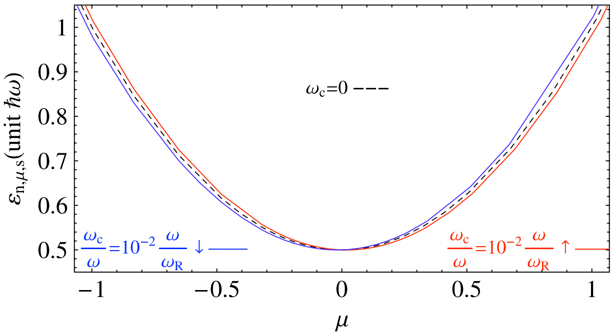

Figure 2. Bandstructure with and without the effects of the SO coupling. Notice the splitting: for each value of the Fermi energy ε_F and for a fixed band n, there are four different eigenvalues. This spin-dependent splitting in the energy allows for the interference phenomena.

while the analogous formulas for the spin currents in the leads are straightforwardly extracted from them [28, 29]

$$I_p^s = \frac{\hbar}{2e}\sum_{q\neq p}(G^{\mathrm{out}}_{qp}V_p - G^{\mathrm{in}}_{pq}V_q), \qquad (9)$$

where $G^{\mathrm{in}}_{pq} = G^{\uparrow\uparrow}_{pq}+G^{\uparrow\downarrow}_{pq}-G^{\downarrow\uparrow}_{pq}-G^{\downarrow\downarrow}_{pq}$ and $G^{\mathrm{out}}_{qp} = G^{\uparrow\uparrow}_{qp}+G^{\downarrow\uparrow}_{qp}-G^{\uparrow\downarrow}_{qp}-G^{\downarrow\downarrow}_{qp}$. The commutation relation $[H,\sigma_z]$ in our system implies $G^{\uparrow\downarrow}_{pq} = G^{\downarrow\uparrow}_{pq} = 0$ so that $G^{\mathrm{in}}_{pq} = G^{\uparrow\uparrow}_{pq} - G^{\downarrow\downarrow}_{pq}$. The conductance coefficients are related to the transmission matrices $\mathbf{t}^{pq}$ between the leads p and q through the Landauer-type formula $G^{\alpha\alpha'}_{pq} = \frac{e^2}{h}\sum_{i,j=1}^{M_{\mathrm{leads}}}|\mathbf{t}^{pq}_{ij,\alpha\alpha'}|^2$, where $|\mathbf{t}^{pq}_{ij,\alpha\alpha'}|^2$ is the probability for a spin-α' electron incident in lead q to be transmitted to lead p as a spin-α electron and i, j label the transverse propagating modes (i.e. conducting channels) in the leads.

Since the total charge current $I_p = I_p^\uparrow + I_p^\downarrow$ depends only on the voltage difference between the leads in figure 1, we set one of them to zero (e.g., $V_3 = 0$ is chosen as the reference potential) and apply the voltage V_1 to the structure. Imposing the requirement $I_2 = I_4 = 0$ for the voltage probes 2 and 4 allows us to get the voltages V_2/V_1 and V_4/V_1 by inverting the multiprobe charge current formulas equation (8). Finally, by solving equation (9) for I_2^s we obtain the most general expression for the spin Hall conductance defined by [29]

$$G_{\mathrm{sH}} = \frac{\hbar}{2e}\frac{I_2^\uparrow - I_2^\downarrow}{V_1 - V_3} = \frac{\hbar}{2e}\left[(G^{\mathrm{out}}_{12}+G^{\mathrm{out}}_{32}+G^{\mathrm{out}}_{42})\frac{V_2}{V_1} - G^{\mathrm{in}}_{42}\frac{V_4}{V_1} - G^{\mathrm{in}}_{21}\right]. \qquad (10)$$

This quantity is measured in units of the spin conductance quantum $e/4\pi$. Moreover, the ballistic four-terminal ring in figure 1 with no impurities has various geometrical symmetries which, together with $G_{pq} = G_{qp}$ property, specify the $V_2/V_1 = V_4/V_1 \equiv 1/2$ solution for the voltages of the transverse leads, when the $I_2 = I_4 = 0$ condition is imposed on their currents [29].

The symmetry of the device is reflected by the transmission coefficients, so that $T^{s,s}_{12} = T^{s,s}_{23} = T^{s,s}_{34} = T^{s,s}_{41}$,

$T_{14}^{s,s} = T_{43}^{s,s} = T_{32}^{s,s} = T_{21}^{s,s}$ and $T_{13}^{s,s} = T_{31}^{s,s}$. The antisymmetric behavior due to the inversion of $B_{\rm eff}$ according the spin polarization gives $T_{12}^{\uparrow\uparrow} = T_{14}^{\downarrow\downarrow}$ and $T_{14}^{\uparrow\uparrow} = T_{12}^{\downarrow\downarrow}$. It follows that

$$G_{\rm sH} = \frac{e}{4\pi} 2\left(T_{12}^{\uparrow\uparrow} - T_{14}^{\uparrow\uparrow}\right), \tag{11}$$

while the corresponding longitudinal charge conductance turns out as

$$G_{\rm L} = \frac{\hbar}{2e}\frac{I_3}{V_1} = \frac{\hbar}{2e}\left[G_{31} + G_{32}\frac{V_2}{V_1} + G_{34}\frac{V_4}{V_1}\right] = \frac{\hbar}{2e}\left(T_{13}^{\uparrow\uparrow} + \frac{T_{12}^{\uparrow\uparrow} + T_{14}^{\uparrow\uparrow}}{2}\right). \tag{12}$$

It is now clear that the nonvanishing $G_{\rm sH}$ stems from the symmetry breaking between the leads 2 and 4 due to the effective magnetic field.

6. Theoretical treatment of the scattering

The study of the conductance is now reduced to the one of finding the transmission coefficients. We approach this scattering problem by using the quantum waveguide theory [30] for the strictly 1D ring and assume the wavevector of the incident propagating electrons in the leads as $k \sim \mu/R_0$. This hypothesis corresponds to assuming the leads as QWs of the same width of the ring. First of all, we introduce the wavefunctions in each of the eight different regions, then we use the Griffith boundary condition [31], which states that the wavefunction is continuous and that the current density is conserved at each intersection between a lead and the ring.

Thus we obtain the transmission coefficients and the related spin conductance $G_{\rm sH}$ reported in figures 3 and 4.

7. Discussion

In this paper we found that a nonvanishing spin Hall conductance can be measured for a four lead ballistic ring just in the presence of the β term of the SO coupling. As we showed in figure 3, some peaks in the pure spin Hall current, $I_{\rm sH}$, are present near the measurable peaks in the longitudinal charge conductance. The presence of a significant $I_{\rm sH}$ is confirmed by the spin spitting of the peaks in the $G_{\rm L}$, as a function of the Fermi energy. All of our calculations are limited to the lowest subband but can be easily extended to the several subband case.

The feasibility of a similar device obviously depends on its size and on the materials. The fundamental theoretical parameter discussed above is the momentum difference, $\Delta\mu$ proportional to the ratio ω_c/ω, corresponding to $\lambda^2/l_\omega^2 = \omega_{\rm eff}/\omega$, i.e. the ratio between a material dependent parameter λ and a size dependent one l_ω (that can be assumed to be a fraction of the real width, W, of the conducting channel). The SO strengths have been theoretically evaluated for some semiconductors compounds. In a QW ($W \sim 100$) patterned in InGaAs/InP heterostructures, where λ^2 takes values between 0.5 and 1.5 nm^2, it gives $\hbar\omega_c \sim 10^{-6}$–$10^{-4}$ eV, corresponding to $\omega_c/\omega \sim 10^{-4}$–$10^{-3}$ as in InSb, where $\lambda^2 \sim 500$ Å^2. For GaAs heterostructures, λ^2 is one order of magnitude

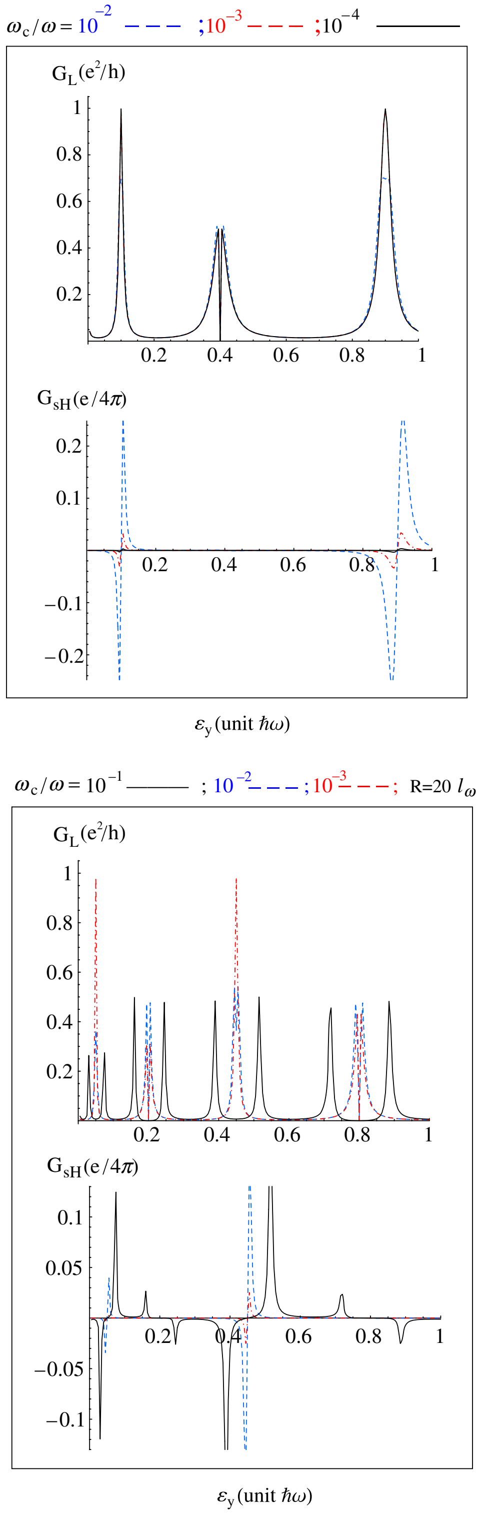

Figure 3. The spin Hall conductance $G_{\rm sH}^z$ (corresponding to the detection of the z-component of the pure spin current I_2^s) for the 1D ring ($R = 10l_\omega$ top panel and $R = 20l_\omega$ bottom panel) attached to four single-channel leads, as a function of the Fermi energy $\varepsilon_{\rm F}$ (bottom panel). We can observe that the presence of peaks in the spin conductance is related to dips in the longitudinal charge transport (upper panel). These dips in $G_{\rm L}$ should correspond to the values of the Fermi energy which give integer angular momenta ($\mu = 1$), but the presence of spin splitting doubles the peak because of the symmetry breaking. By comparing the top panel and the bottom panel, it is clear that the width of the peaks strongly depends on the radius of the ring.

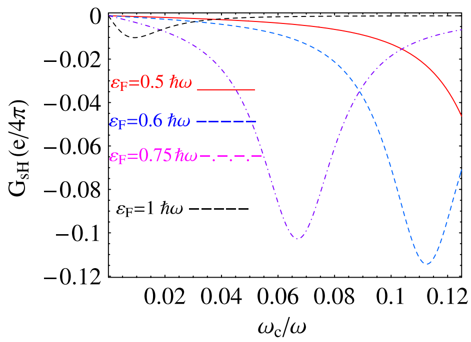

Figure 4. The spin Hall conductance G^z_{sH} for the 1D ring ($R = 10l_\omega$) attached to four single-channel leads, as a function of dimensionless SO coupling ω_c/ω.

smaller (∼4.4 Å^2) than in InGaAs/InP, whereas for HgTe based heterostructures it can be more than three times as large [32]. However, the lithographical width of a wire defined in a 2DEG can be as small as 20 nm [33]; thus we can realistically assume that ω_c/ω runs from 1×10^{-6} to 1×10^{-1}.[3]

Acknowledgment

We acknowledge the support of the grant 2006 PRIN 'Sistemi Quantistici Macroscopici-Aspetti Fondamentali ed Applicazioni di strutture Josephson Non Convenzionali'.

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[3] In any case W should be larger than λ_F, so that at least one conduction mode is occupied.